EX 1 :

1) $\int_{-1}^{3}(\frac{x^2}{2}-x)dx=\left[\frac{1}{6}x^3-\frac{1}{2}x^2\right]_{-1}^{3}=\frac{2}{3}$

2) $\int_{+1}^{2}t.(t+1)^3.dt=\int_{1}^{2}(t^4+3t^3+3t^2+t).dt$

$=\left[\frac{t^5}{5}+\frac{3t^4}{4}+t^3+\frac{t^2}{2}\right]_1^2=\frac{519}{20}$

3) $\int_{-1}^{1}(2x+1).(x^2+x-5)^3dx=\left[\frac{1}{4}(x^2+x-5)^4\right]_{-1}^{1}$

$=\frac{3^4}{4}-\frac{5^4}{4}=\frac{-272}{2}=-136$

4) $\int_{1}^{2}\sqrt{2x+1}dx=\frac{1}{2}\int_{1}^{2}2.\sqrt{2x+1}dx$

$=\frac{1}{2}.\left[\frac{2}{3}(2x+1)\sqrt{2x+1}\right]_1^2=\frac{1}{3}(5\sqrt{5}-3\sqrt{3})$

5) $\int_{1}^{2}\frac{dx}{(1+x)^2}=\left[\frac{-1}{1+x}\right]_1^2=\frac{1}{6}$

6) $\int_{0}^{4}\frac{dx}{\sqrt{2x+1}}=\int_{0}^{4}\frac{2}{2\sqrt{2x+1}}dx$

$=\left[\sqrt{2x+1}\right]_0^4=2$

7) $\int_{0}^{\frac{\pi}{4}}\sin t.\cos^4 tdt=\left[\frac{-1}{5}\cos^5 t\right]_0^{\frac{\pi}{4}}=\frac{8-\sqrt{2}}{40}$

8) $\int_{-2}^{1}|x(x+1)|.dx=I$

x	-2	-1	0	1
x(x+1)	+	0 -	0	+

$I=\int_{-2}^{-1}x(x+1)dx+\int_{-1}^{0}-x(x+1)dx+\int_{0}^{1}x(x+1)dx$

$I=\left[\frac{x^3}{3}+\frac{x^2}{2}\right]_{-2}^{-1}-\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_{-1}^{0}-\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_{0}^{1}=\frac{5}{3}$

9) $x^3+x=x.(x^2+1)$

$x\mapsto|x^3+x|$ est paire

$\int_{-3}^{3}|x^3+x|dx=2\int_{0}^{3}|x^3+x|dx=2\int_{0}^{3}(x^3+x)dx$

$=2.\left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^3=\frac{99}{2}$

10)

t	0	½	3
2t-1	-	0	+

$\int_{0}^{3}|2t-1|dt=\int_{0}^{\frac{1}{2}}(-2t+1)dt+\int_{\frac{1}{2}}^{3}(2t-1)dt$

$=\left[-t^2+t\right]_0^{\frac{1}{2}}+\left[t^2-t\right]_{\frac{1}{2}}^{3}=\frac{13}{2}$

11) $\int_{0}^{2\pi}|\sin x|dx=\int_{0}^{\pi}|\sin x|dx+\int_{\pi}^{2\pi}|\sin x|dx$

$=\int_{0}^{\pi}\sin xdx-\int_{\pi}^{2\pi}\sin xdx$

$=\left[-\cos x\right]_0^{\pi}+\left[\cos x\right]_{\pi}^{2\pi}=4$

EX 2 :

$aire(D_1\cup D_2)=2\int_{0}^{2}(4-x^2)dx$

$=2\left[4x-\frac{x^3}{3}\right]_0^2=\frac{32}{3}$

$aire(D_1)=2\int_{0}^{\sqrt{a}}(a-x^2)dx=2\left[ax-\frac{x^3}{3}\right]_0^{\sqrt{a}}$

$=2(a\sqrt{a}-\frac{a\sqrt{a}}{3})=\frac{4a\sqrt{a}}{3}$

$aire(D_1)=aire(D_2)$

$\Leftrightarrow aire(D_1)=\frac{1}{2}aire(D_1\cup D_2)$

$\Leftrightarrow\frac{4a\sqrt{a}}{3}=\frac{32}{6}\Leftrightarrow a\sqrt{a}=4\Leftrightarrow(\sqrt{a})^3=4$

$\Leftrightarrow\sqrt{a}=\sqrt[3]{4}\Leftrightarrow a=(\sqrt[3]{4})^2\Leftrightarrow a=\sqrt[3]{16}=2\sqrt[3]{2}$

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